DEPENDENCY STRUCTURE MISSPECIFICATION IN MULTI-SOURCE WEAK SUPERVISION MODELS

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ABSTRACT

Data programming (DP) has proven to be an attractive alternative to costly hand-labeling of data. In DP, users encode domain knowledge into *labeling functions* (LF), heuristics that label a subset of the data noisily and may have complex dependencies. A label model is then fit to the LFs to produce an estimate of the unknown class label. The effects of label model misspecification on test set performance of a downstream classifier are understudied. This presents a serious awareness gap to practitioners, in particular since the dependency structure among LFs is frequently ignored in field applications of DP. We analyse modeling errors due to structure over-specification. We derive novel theoretical bounds on the modeling error and empirically show that this error can be substantial, even when modeling a seemingly sensible structure.

1 INTRODUCTION

Annotating large datasets for machine learning is expensive, time consuming, and a bottleneck for many practical applications of artificial intelligence. Recently, data programming, a paradigm that makes use of multiple weak supervision sources, has emerged as a promising alternative to manual data annotation (Ratner et al., 2016). In this framework, users encode domain knowledge into weak supervision sources, such as domain heuristics or knowledge bases, that each noisily label a subset of the data. A generative model over the sources and the latent true label is then learned. One can use the learned model to estimate *probabilistic* labels to train a *downstream model*, replacing the need to obtain ground truth labels by manual annotation of individual samples.
In practice, the sources of weak labels that users define often exhibit statistical dependencies amongst each other, e.g. sources operating on the same or similar input (some examples can be found in Table 1). Defining the correct dependency structure is difficult, thus a common approach in popular libraries (Bach et al., 2019; Ratner et al., 2019a) and related research (Dawid & Skene, 1979; Anandkumar et al., 2014; Varma & Ré, 2018; Boecking et al., 2021) is to ignore it. However, the implications of this assumption on downstream performance have not been researched in detail. Therefore, in this paper we take steps towards gaining a better understanding of the trade-offs involved.

Contributions We present novel bounds on the label model posterior and the downstream generalization risk that are explicitly influenced by misspecified higher-order dependencies. We also introduce three new higher-order dependency types, which we name *bolstering*, *negated*, and *priority* dependencies. Lastly, we empirically show that downstream test performance is highly sensitive to the user-specified dependencies, even when they make sense semantically. The finding suggests that in practice, it is advisable to only carefully model a few, if any, dependencies.

2 RELATED WORK

Data programming While the original data programming framework (Ratner et al., 2016) is based on a factor graph that support the modeling of arbitrary dependencies between LFs, more recent methods for solving for the parameters of the label model only support the modeling of pairwise correlations (Ratner et al., 2019a;b; Fu et al., 2020) — as such, losing some of the expressive power of data programming. The former extends data programming to the multi-task setting by exploiting the graph structure of the inverse covariance matrix among the sources (Ratner et al., 2019b) — in particular the fact that an entry is zero when there is no edge between the corresponding sources in the graphical model (Loh & Wainwright, 2012). The latter finds a closed-form solution for a class of binary Ising models by using triplet methods (Fu et al., 2020). Our experimental findings suggest that practitioners may indeed benefit from simply ignoring higher-order dependencies.

Structure learning In order to automatically learn the structure between these sources, previous work optimizes the marginal pseudolikelihood of the noisy labels (Bach et al., 2017), or makes use of robust PCA to denoise the inverse covariance matrix of the sources labels into a graph structured term (Varma et al., 2019). A different approach, infers the structure through static analysis of the weak supervision sources code definitions and thereby reduces the sample complexity for learning the structure (Varma et al., 2017). In our experiments, we show that such methods should be carefully used, and may in fact lead to downstream performance losses.

Model misspecification On the side of work on model misspecification, (White, 1982) establishes that the Maximum Likelihood Estimator of a misspecified model is a consistent estimator of the learnable parameter that minimizes the KL divergence to the true distribution – if that optimal, misspecified parameter is globally identifiable. In an interesting result, (Jog & Loh, 2015) show that the KL divergence between a multivariate Gaussian distribution and a misspecified (by at least a single edge) Gaussian graphical model is bounded by a constant from below. It emphasizes the need to correctly select the model's edge structure, since otherwise the fitted distribution will differ from the true one in terms of KL divergence.

3 Problem setup

For this work we use the data programming framework as introduced in (Ratner et al., 2016), where the premise is that experts can model any higher-order dependency between labeling functions. We extend it by *negated*, *bolstering*, *priority* dependencies (definitions in the appendix B), e.g. the latter encoding the notion that one source's vote should be prioritized over the one from a noisier source. The additional dependency types are motivated by our selected LF sets that contain dependencies, such as the ones in Table 1, that we could not express before. Newer weak supervision models and model fitting approaches often only allow for pairwise correlation dependencies to be modeled (Ratner et al., 2019b; Fu et al., 2020).

Let $(x, y) \sim \mathcal{D}$ be the true data generating distribution and for simplicity assume that $y \in \{-1, 1\}$. As in (Ratner et al., 2016), users provide m labeling functions (LFs) $\lambda = \lambda(x) \in \{-1, 0, 1\}^m$, where 0 means that the LF abstained from labeling. Following (Ratner et al., 2016), we model the joint distribution of y, λ as a factor graph. To study model misspecification we compare two label models, p_θ for the conditional independent case and p_μ which models M higher-order dependencies:

$$p_\mu(\lambda, y) = \frac{1}{Z_\mu} \exp\left(\mu^T \phi(\lambda, y)\right) = Z_\mu^{-1} \exp\left(\mu_1^T \phi_1(\lambda, y) + \mu_2^T \phi_2(\lambda, y)\right), \qquad \mu \in \mathbb{R}^{m+M} \quad (1)$$

$$p_\theta(\lambda, y) = Z_\theta^{-1} \exp\left(\theta^T \phi_1(\lambda, y)\right), \qquad \theta \in \mathbb{R}^m, \quad (2)$$

where $\phi_1(\lambda, y) = \lambda y$ are the accuracy factors, $\phi_2(\cdot)$ are arbitrary, higher-order dependencies and $Z_\theta^{-1}, Z_\mu^{-1}$ are normalization constants. We assume w.l.o.g. that factors are bounded ≤ 1. Note that for ease of exposition, the models above do not model the labeling propensity factor (also known as LF coverage) . Since this factor does not depend on the label, the corresponding terms would cancel out in the quantities studied below (same goes for dependencies that do not depend on y, e.g. the *similar* factor from (Ratner et al., 2016)).

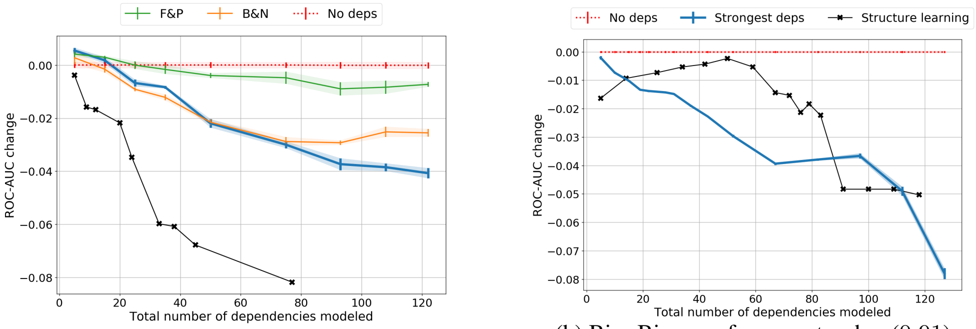
4 Theoretical analysis

Bound on the label model posterior under model misspecification We now state our bound on the probabilistic label difference (which we prove in the appendix A):

$$|p_\mu(y \mid \lambda) - p_\theta(y \mid \lambda)| \leq \frac{1}{2} ||\mu_1 - \theta||_1 + \frac{1}{4} ||\mu_2||_1 \quad (3)$$

This is an important quantity of interest since the probabilistic labels are used to train a downstream model. Unsurprisingly then, this quantity reappears as a main factor controlling the KL divergence and generalization risk, see below.

Suppose that the downstream model $f_w : \mathcal{X} \to \mathcal{Y}$ is parameterized by w, and that to learn w we minimize a, w.l.o.g., bounded noise-aware loss function $L(f_w(x), y) \in [0, 1]$ (that acts on the probabilistic labels). If we had access to the true labels, we would normally try to find the w that minimizes the risk: $w^* = \arg\min_w R(w) = \arg\min_w \mathbb{E}_{(x,y)\sim D} [L(f_w(x), y)]$. Since this is not the case, we instead will get the parameter $\hat{w}$ that minimizes the (empirical) noise-aware loss.



(a) IMDB (0.81)

(b) Bias Bios: professor or teacher (0.91)

Figure 1: Modeling more than a handful dependencies (as the ones in Table 1) significantly deteriorates ROC-AUC downstream performance as compared to simply ignoring them by up to 4 and 8 points. This effect intensifies as we model more dependencies. In brackets, the baseline score for the independent model ('No deps'). `B&N` means that we only model the *bolstering* and *negated* dependencies, while `F&P` means that we only model the *fixing* and *priority* dependencies. `Structure learning` means that we model the *similar*, *fixing* and *reinforcing* dependencies returned by the method from (Bach et al., 2017) for different threshold hyperparameters (the lower, the more dependencies are modeled).

Bound on the generalization risk under model misspecification While (Fu et al., 2020) provide a bound on the generalization error that accounts for model misspecification – the label model being a less expressive Ising model (i.e. no higher-order dependencies may be modeled) – the part of the bound corresponding to the misspecification is a somewhat loose KL-divergence term between the true and the misspecified models. If we instead assume that there exists a label model for some optimal parameterization of factors (not necessarily the ones that are actually modeled) that is equivalent to the true data generating distribution, we can provide a more meaningful and tight bound.

As in (Ratner et al., 2016; 2019b), assume that 1) there exists an optimal parameter of either p_μ or p_θ (say, μ^* with label model p_{μ^*}) such that sampling (λ, y) from this optimal label model is equivalent to $(\lambda, y) \sim \mathcal{D}$; and 2) the label y is independent of the features used for training f_w given the labeling function outputs λ. Differently than (Ratner et al., 2016; 2019b), the factors that are actually modeled may differ from the ones of the optimal label model. By adapting the proof of theorem 1 in (Ratner et al., 2019b) to our case where we incorrectly use a misspecified model (say, p_θ), we can bound the generalization risk as follows

$$R(\hat{w}) - R(w^*) \leq \gamma + 2||\mu_1^* - \theta||_1 + ||\mu_2^*||_1, \tag{4}$$

where γ is a bound on the empirical risk minimization error, which is not specific to our setting. We can bound the KL divergence of the two models by a similar quantity, see the appendix A.3.

Interpretation These bounds naturally involve the accuracy parameter estimation error $||\mu_1 - \theta||_1$ and the learned strength $s := ||\mu_2||_1$ of the dependencies only modeled in p_μ. Note that if we assume that the model with dependencies p_μ is the true one we can interpret s as the magnitude of the dependencies that the independent model p_θ failed to model. If on the other hand we associate the misspecified model with p_μ, then we can interpret this quantity s as the (dependency) parameter excess that was incorrectly learned by p_μ. The presented bounds are tighter than the ones from (Ratner et al., 2019b) for the L1 norm, while in addition accounting for model misspecification.

5 Experiments

5.1 Proxy for finding true dependencies in real datasets

The underlying *true* structure between two real labeling functions λ_j, λ_k is, of course, unknown. However, by using true training labels (solely for this purpose) together with the observed LF votes, we can compute resulting factor values for each data point i, to observe empirical strength of dependency factor l over a training set: $v^l_{j,k} = \sum_i \phi^l(\lambda(x_i)_j, \lambda(x_i)_k, y_i)$. Sorting dependencies l

Table 1: The strongest and weakest dependencies among the IMDB LFs, as measured by their factor value $v^l_{j,k}$ computed with respect to the true labels. In the experiment from Fig.1, we repeatedly add weaker dependencies to the set of dependencies used for learning the label model, starting with the strongest ones below.

LF$_j$	LF$_k$	factor type l	factor value $v^l_{j,k}$
best	great	bolstering	801
bad	don’t waste	bolstering	110
original	bad	priority	327
recommend	terrible	priority	53
worth	not worth	fixing	238
great	nothing great	fixing	15
worth	not worth	negated	219
special	not special	negated	8
recommend	highly recommend	reinforcing	226
bad	absolutely horrible	reinforcing	7

according to $v^l_{j,k}$ in descending order, we then choose to model the top d dependencies. These are the dependencies for which the true labels provide the most evidence of being correct.

5.2 Performance deterioration due to structure over-specification

For the following experiment we use the IMDB Movie Review Sentiment dataset consisting of $n = 25$k training and test samples each (Maas et al., 2011) and manually select a set of $m = 135$ sensible LFs that label on the presence of a single word or a pair of words (i.e. uni-/bi-gram LFs). In addition we use the Bias in Bios dataset (De-Arteaga et al., 2019), and focus on the binary classification task introduced in (Boecking et al., 2021) that aims at distinguishing the biographies of professors and teachers ($n = 12294, m = 85$). We deliberately choose unigram and bigram LFs so as to create dependencies we expect to help with downstream model performance, e.g. by adding negations (e.g. ”not worth”) of unigrams (e.g. ”worth”) that we expect to fix the less precise votes of the latter when both do not abstain.

We choose different $d \in \{1, 3, 5, \ldots, 40\}$ and then model the strongest $\leq d$ dependencies of each factor l according to v^l. An example of the strongest and weakest dependencies for the IMDB dataset is shown in Table 1. For the Bias in Bios experiment, an example of a strong bolstering dependency is that the term ‘PhD’ appears in addition to the term ‘university’. We report the test set performance of a simple 3-layer neural network trained on the probabilistic labels, averaged out over 100 runs. While for IMDB we observe a marginal boost (< 0.005) in performance when modeling the strongest $d = 1, 3$ dependencies of each factor ($5, 15$ in total), the main take-away is the following:

We find that modeling more than a handful of dependencies significantly *deteriorates* the downstream end classifier performance (by up to 8 ROC-AUC points) as compared to simply ignoring them (Fig. 1). The performance worsens as we increase d, i.e. as we model more, slightly weaker, dependencies. We reiterate that these additional dependencies still, semantically, make sense (as depicted in Table 1, where the weakest ones are modeled only for the case where the total number of dependencies $= 122$). Using the structure learning method from (Bach et al., 2017) to infer the dependency structure results in worse test performances too.

6 Discussion and Future Work

Discussion Even though this result and insight is highly relevant for practitioners, it has, to the best of our knowledge, not been explored in detail. It may come as a surprise that modeling seemingly sensible dependencies can significantly deteriorate the targeted downstream model performance. We hypothesize that this is due to the true model being close to the conditionally independent case and in our presented bounds, we indeed see that they become looser as more incorrect dependencies are modeled. In addition, more complex models often suffer of a higher sample complexity, as is also briefly noted in (Fu et al., 2020). This suggests that practitioners may 1) indeed be best served, at first, by simply ignoring (higher-order) dependencies; and 2) need to be careful when specifying dependencies, either by hand or through structure learning algorithms, which emphasizes the need for a small ground-truth labeled validation set to compare the performance of different label models.

Future work Future work should give a theoretically precise answer to the reason for why performance deterioration is observed, conduct more extensive experiments to validate that this holds for a variety of datasets and labeling function sets, as well as better characterize the settings in which structure learning actually helps with downstream performance.

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A PROOFS OF THE THEORETICAL ANALYSIS

A.1 PROBLEM SETUP RECAP

Let $(x, y) \sim \mathcal{D}$ be the true data generating distribution and for simplicity assume that $y \in \mathcal{Y} = \{-1, 1\}$. Users provide m labeling functions (LFs) $\lambda = \lambda(x) \in \{-1, 0, 1\}^m$, where 0 means that the LF abstained from labeling. We compare two label models, p_θ for the conditional independent case, and p_μ which models higher-order dependencies:

$$p_\mu(\lambda, y) = \frac{1}{Z_\mu} \exp\left(\mu^T \phi(\lambda, y)\right) = Z_\mu^{-1} \exp\left(\mu_1^T \phi_1(\lambda, y) + \mu_2^T \phi_2(\lambda, y)\right), \qquad \mu \in \mathbb{R}^{m+M} \tag{5}$$

$$p_\theta(\lambda, y) = Z_\theta^{-1} \exp\left(\theta^T \phi_1(\lambda, y)\right), \qquad \theta \in \mathbb{R}^m, \tag{6}$$

where $\phi_1(\lambda, y) = \lambda y$ are the accuracy factors, $\phi_2(\cdot)$ are arbitrary, higher-order dependencies and $Z_\theta^{-1}, Z_\mu^{-1}$ are normalization constants. We assume w.l.o.g. that factors are bounded ≤ 1. Using an unlabeled dataset $X = \{x_i\}_{i=1}^n$ of n data points $x_i \in \mathcal{X}$ to which we each apply the m user provided labeling functions, we attain the label matrix $\Lambda \in \{-1, 0, 1\}^{n \times m}$. With Λ we then train the label model to get a set of n probabilistic labels with which we supervise the downstream model $f_w : \mathcal{X} \to \mathcal{Y}$.

Lemma 1 (Sigmoid posterior). With $\sigma(x) = \frac{1}{1+\exp(-x)}$ being the sigmoid function, it holds that

$$p_\mu(y \mid \lambda) = \sigma\left(2\mu_1^T \phi_1(\lambda, y) + \mu_2^T \left(\phi_2(\lambda, y) - \phi_2(\lambda, -y)\right)\right) \tag{7}$$

$$p_\theta(y \mid \lambda) = \sigma\left(2\theta^T \phi_1(\lambda, y)\right). \tag{8}$$

Proof

$$\begin{aligned}
p_\mu(y \mid \lambda) &= \frac{p_\mu(\lambda, y)}{p_\mu(\lambda)} = \frac{p_\mu(\lambda, y)}{\sum_{\tilde{y} \in \mathcal{Y}} p_\mu(\lambda, \tilde{y})} \\
&= \frac{Z_\mu^{-1} \exp\left(\mu^T \phi(\lambda, y)\right)}{\sum_{\tilde{y} \in \mathcal{Y}} Z_\mu^{-1} \exp\left(\mu^T \phi(\lambda, \tilde{y})\right)} \\
&= \frac{\exp\left(\mu^T \phi(\lambda, y)\right)}{\sum_{\tilde{y} \in \mathcal{Y}} \exp\left(\mu^T \phi(\lambda, \tilde{y})\right)} \\
&= \frac{\exp\left(\mu^T \phi(\lambda, y)\right)}{\exp\left(\mu^T \phi(\lambda, y)\right) + \exp\left(\mu^T \phi(\lambda, -y)\right)} \\
&= \frac{1}{1 + \exp\left(\mu^T \left(\phi(\lambda, -y) - \phi(\lambda, y)\right)\right)} \\
&= \sigma\left(\mu^T \left(\phi(\lambda, y) - \phi(\lambda, -y)\right)\right) \\
&= \sigma\left(2\mu_1^T \phi_1(\lambda, y) + \mu_2^T \left(\phi_2(\lambda, y) - \phi_2(\lambda, -y)\right)\right),
\end{aligned}$$

where in the last step we used the fact that the accuracy factors are odd functions, i.e. $\phi_1(\lambda, -y) = -\lambda y = -\phi_1(\lambda, y)$. Eq. 8 follows by the same argumentation.

A.2 Proof of the bound on the label model posterior

Bound Our bound on the probabilistic label difference between the two models above is:

$$|p_\mu(y\,|\lambda) - p_\theta(y\,|\lambda)| \leq \frac{1}{2}||\mu_1 - \theta||_1 + \frac{1}{4}||\mu_2||_1 \tag{9}$$

Proof Using Lemma 1 we have that

$$|p_\mu(y\,|\lambda) - p_\theta(y\,|\lambda)| = \left|\sigma\left(2\mu_1^T\phi_1(\lambda,y) + \mu_2^T\left(\phi_2(\lambda,y) - \phi_2(\lambda,-y)\right)\right) - \sigma\left(2\theta^T\phi_1(\lambda,y)\right)\right|$$

By the mean value theorem it follows that for some c between the arguments of σ above

$$\begin{aligned}&= \sigma'(c)\left|\left(2\mu_1^T\phi_1(\lambda,y) + \mu_2^T\left(\phi_2(\lambda,y) - \phi_2(\lambda,-y)\right)\right) - 2\theta^T\phi_1(\lambda,y)\right| \\ &= \sigma'(c)\left|2\left(\mu_1 - \theta\right)^T\phi_1(\lambda,y) + \mu_2^T\left(\phi_2(\lambda,y) - \phi_2(\lambda,-y)\right)\right|\end{aligned}$$

Using the triangle inequality and the fact that $\max_x \sigma'(x) = \max_x \sigma(x)(1-\sigma(x)) = \frac{1}{4}$, we can now bound this expression as follows

$$\leq \frac{1}{2}\left|\left(\mu_1 - \theta\right)^T\phi_1(\lambda,y)\right| + \frac{1}{4}\left|\mu_2^T\left(\phi_2(\lambda,y) - \phi_2(\lambda,-y)\right)\right|$$

finally, since the defined higher-order dependencies are indicator functions $\neq 0$ for only one $y \in \mathcal{Y}$, and if $||q||_\infty \leq 1$ then $|x^Tq| = |\sum_i x_iq_i| \leq \sum_i |x_iq_i| \leq \sum_i |x_i| = ||x||_1$, this reduces to

$$\leq \frac{1}{2}||\mu_1 - \theta||_1 + \frac{1}{4}||\mu_2||_1.$$

A.3 Proof of the bound on the KL divergence

Bound

$$KL\left(p_\mu(y\,|\lambda)\;||\;p_\theta(y\,|\lambda)\right) \leq 2||\mu_1 - \theta||_1 + ||\mu_2||_1 \tag{10}$$

Proof We adapt Theorem 7 of (Honorio, 2011) to give a bound on the KL divergence between the two label model posterior's. First note that with the same line of argumentation as in A.2 we have that

$$\begin{aligned}|\log p_\mu(y\,|\lambda) - \log p_\theta(y\,|\lambda)| &= (\log\sigma)'(c)\left|2\left(\mu_1 - \theta\right)^T\phi_1(\lambda,y) + \mu_2^T\left(\phi_2(\lambda,y) - \phi_2(\lambda,-y)\right)\right| \\ &\leq 2||\mu_1 - \theta||_1 + ||\mu_2||_1,\end{aligned}$$

where we use that the derivative of $\log\sigma(\cdot)$ is $(1+\exp(x))^{-1} \in (0,1)$, and is bounded by 1. Next

$$\begin{aligned}KL\left(p_\mu(y\,|\lambda)\;||\;p_\theta(y\,|\lambda)\right) &= \sum_{\lambda\in L} p_\mu(\lambda)\sum_{y\in\mathcal{Y}} p_\mu(y|\lambda)\cdot\log\left(\frac{p_\mu(y|\lambda)}{p_\theta(y|\lambda)}\right) \\ &= \sum_{\lambda\in L} p_\mu(\lambda)\sum_{y\in\mathcal{Y}} \frac{p_\mu(\lambda,y)}{p_\mu(\lambda)}\cdot\log\left(\frac{p_\mu(y|\lambda)}{p_\theta(y|\lambda)}\right) \\ &= \sum_{\lambda\in L}\sum_{y\in\mathcal{Y}} p_\mu(\lambda,y)\cdot\left(\log p_\mu(y|\lambda) - \log p_\theta(y|\lambda)\right) \\ &\leq \sum_{\lambda\in L}\sum_{y\in\mathcal{Y}} p_\mu(\lambda,y)\cdot\left|\log p_\mu(y|\lambda) - \log p_\theta(y|\lambda)\right| \\ &\leq \left(2||\mu_1 - \theta||_1 + ||\mu_2||_1\right)\sum_{\lambda\in L}\sum_{y\in\mathcal{Y}} p_\mu(\lambda,y) \\ &= 2||\mu_1 - \theta||_1 + ||\mu_2||_1\end{aligned}$$

A.4 Proof of the generalization risk bound

We now adapt Theorem 1 from (Ratner et al., 2019b) to the setting with model misspecification and assume like in (Ratner et al., 2016; 2019b) that

1. there exists an optimal parameter of either p_μ or p_θ such that sampling (λ, y) from this optimal label model is equivalent to $(\lambda, y) \sim \mathcal{D}$.
2. the label y is independent of the features used for training f_w given the labeling function outputs λ, i.e. the LF labels provide sufficient signal to identify the label.

For 1. we now assume without loss of generality, that $p_{\mu^*}(\lambda, y) = p_{\mathcal{D}}(\lambda, y)$ for an optimal parameter $\mu^* \in \mathbb{R}^{m+M}$, and that we incorrectly use the misspecified label model p_θ. For the reversed roles (i.e. p_θ is the true model and p_μ is misspecified), the following arguments are symmetric.
Suppose that the downstream model $f_w : \mathcal{X} \to \mathcal{Y}$ is parameterized by w, and that to learn w we minimize a, w.l.o.g., bounded loss function $L(f_w(x), y) \in [0, 1]$. If we had access to the true labels, we would normally try to find the w that minimizes the risk:

$$w^* = \arg\min_w R(w) = \arg\min_w \mathbb{E}_{(x,y)\sim D}\left[L(f_w(x), y)\right]. \tag{11}$$

Since this is not the case, we instead minimize the noise-aware loss:

$$\tilde{w} = \arg\min_w R_\theta(w) = \arg\min_w \mathbb{E}_{(x,y)\sim \mathcal{D}}\left[\mathbb{E}_{\tilde{y}\sim p_\theta(\cdot|\lambda)}\left[L(f_w(x), \tilde{y})\right]\right]. \tag{12}$$

In practice we will get the parameter $\hat{w}$ that minimizes the empirical noise-aware loss over the unlabeled dataset $X = \{x_1, ..., x_n\}$:

$$\hat{w} = \arg\min_w \hat{R}_\theta(w) = \arg\min_w \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}_{\tilde{y}\sim p_\theta(\cdot|\lambda(x_i))}\left[L(f_w(x_i), \tilde{y})\right]. \tag{13}$$

Since the empirical risk minimization error is not specific to our setting and can be done with standard methods, we simply assume that the error $|R_\theta(\tilde{w}) - R_\theta(\hat{w})| \le \gamma(n)$, where $\gamma(n)$ is a decreasing function of the unlabeled dataset size n.

Bound By adapting the proof of theorem 1 in (Ratner et al., 2019b) to our case with model misspecification involved, we can bound the generalization risk as follows

$$R(\hat{w}) - R(w^*) \le \gamma(n) + 2||\mu_1^* - \theta||_1 + ||\mu_2^*||_1. \tag{14}$$

Note that the bound from (Ratner et al., 2019b) is mistakenly too tight by a factor of 2 (due to the last step in the proof below that is partly overseen).

Proof First, using the law of total expectation, followed by our assumptions 2. and 1., in that order, we have that:

$$\begin{aligned}
R(w) &= \mathbb{E}_{(x',y')\sim D}\left[R(w)\right] \\
&= \mathbb{E}_{(x',y')\sim D}\left[\mathbb{E}_{(x,y)\sim D}\left[L(f_w(x'), y)|x = x'\right]\right] \\
&= \mathbb{E}_{(x',y')\sim D}\left[\mathbb{E}_{(x,y)\sim D}\left[L(f_w(x'), y)|\lambda(x) = \lambda(x')\right]\right] \\
&= \mathbb{E}_{(x',y')\sim D}\left[\mathbb{E}_{\tilde{y}\sim p_{\mu^*}(\cdot|\lambda)}\left[L(f_w(x'), \tilde{y})\right]\right] \\
&= R_{\mu^*}(w)
\end{aligned}$$

Using the result above that $R = R_{\mu^*}$ and adding and subtracting terms we have:

$$\begin{aligned}
R(\hat{w}) - R(w^*) &= R_{\mu^*}(\hat{w}) - R_{\mu^*}(w^*) \\
&= R_{\mu^*}(\hat{w}) + R_\theta(\hat{w}) - R_\theta(\hat{w}) + R_\theta(\tilde{w}) - R_\theta(\tilde{w}) - R_{\mu^*}(w^*)
\end{aligned}$$

since $\tilde{w}$ minimizes the noise-aware risk, i.e. $R_\theta(\tilde{w}) \le R_\theta(w^*)$, we have that:

$$\begin{aligned}
&\le R_{\mu^*}(\hat{w}) + R_\theta(\hat{w}) - R_\theta(\hat{w}) + R_\theta(w^*) - R_\theta(\tilde{w}) - R_{\mu^*}(w^*) \\
&\le |R_\theta(\hat{w}) - R_\theta(\tilde{w})| + |R_{\mu^*}(\hat{w}) - R_\theta(\hat{w})| + |R_\theta(w^*) - R_{\mu^*}(w^*)| \\
&\le \gamma(n) + 2\max_{w'} |R_{\mu^*}(w') - R_\theta(w')|
\end{aligned}$$

The main term $|R_{\mu^*}(w') - R_\theta(w')|$ we now have in the bound, is the difference between the true/expected noise-aware losses given the true label model parameter μ^* and the estimated parameter

θ for the misspecified model. We now bound this quantity:

$$\begin{aligned}
|R_{\mu^*}(w') - R_\theta(w')| &= \left|\mathbb{E}_{(x,y)\sim\mathcal{D}}\left[\mathbb{E}_{\tilde{y}\sim p_{\mu^*}(\cdot|\lambda)}\left[L(f_w(x),\tilde{y})\right] - \mathbb{E}_{\tilde{y}\sim p_\theta(\cdot|\lambda)}\left[L(f_w(x),\tilde{y})\right]\right]\right| \\
&= \left|\mathbb{E}_{(x,y)\sim\mathcal{D}}\left[\sum_{y'\in\mathcal{Y}} L(f_w(x),y')\left(p_{\mu^*}(y' \mid \lambda) - p_\theta(y' \mid \lambda)\right)\right]\right| \\
&\leq \sum_{y'\in\mathcal{Y}} \mathbb{E}_{(x,y)\sim\mathcal{D}}\left[|L(f_w(x),y')\left(p_{\mu^*}(y' \mid \lambda) - p_\theta(y' \mid \lambda)\right)|\right] \\
&\leq \sum_{y'\in\mathcal{Y}} \mathbb{E}_{(x,y)\sim\mathcal{D}}\left[|p_{\mu^*}(y' \mid \lambda) - p_\theta(y' \mid \lambda)|\right] \\
&\leq |\mathcal{Y}| \max_{y'} \mathbb{E}_{(x,y)\sim\mathcal{D}}\left[|p_{\mu^*}(y' \mid \lambda) - p_\theta(y' \mid \lambda)|\right]
\end{aligned}$$

We can now use our bound from (3) and get that:

$$\leq 2\left(\frac{1}{2}||\mu_1^* - \theta||_1 + \frac{1}{4}||\mu_2^*||_1\right) = ||\mu_1^* - \theta||_1 + \frac{1}{2}||\mu_2^*||_1$$

Plugging this back into the term for the generalization risk gives the desired result:

$$\begin{aligned}
R(\hat{w}) - R(w^*) &\leq \gamma(n) + 2\max_{w'}|R_{\mu^*}(w') - R_\theta(w')| \\
&\leq \gamma(n) + 2||\mu_1^* - \theta||_1 + ||\mu_2^*||_1.
\end{aligned}$$

B Factor Definitions

We supplement the factor definitions of higher-order dependencies used in this paper. The first two stem from (Ratner et al., 2016), the rest we defined ourselves for the conducted experiments, and where motivated by frequently occurring dependency patterns, as the ones in Table 1, that are not covered by (Ratner et al., 2016). Whenever a factor $\phi_{j,k}(\lambda, y)$ is not symmetric (all factors, besides *bolstering*), we define it so that LF$_k$ acts on (e.g. *negates*) LF$_j$.
For the *fixing* dependency we have:

$$\phi_{j,k}^{Fix}(\lambda, y) = \begin{cases} +1 & \text{if } \lambda_j = -y \wedge \lambda_k = y \\ -1 & \text{if } \lambda_j = 0 \wedge \lambda_k \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

for the *reinforcing* one:

$$\phi_{j,k}^{Rei}(\lambda, y) = \begin{cases} +1 & \text{if } \lambda_j = \lambda_k = y \\ -1 & \text{if } \lambda_j = 0 \wedge \lambda_k \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

for the *priority* factor:

$$\phi_{j,k}^{Pri}(\lambda, y) = \begin{cases} +1 & \text{if } \lambda_j = -y \wedge \lambda_k = y \\ -1 & \text{if } \lambda_j = y \wedge \lambda_k = -y \\ 0 & \text{otherwise} \end{cases}$$

for the *bolstering*:

$$\phi_{j,k}^{Bol}(\lambda, y) = \begin{cases} +1 & \text{if } \lambda_j = \lambda_k = y \\ -1 & \text{if } \lambda_j = \lambda_k \neq y \vee \lambda_j = -\lambda_k \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

and, finally, for the *negated* factor:

$$\phi_{j,k}^{Neg}(\lambda, y) = \begin{cases} +1 & \text{if } \lambda_j = -y \wedge \lambda_k = y \\ -1 & \text{if } (\lambda_j = y \wedge \lambda_k = -y) \vee \lambda_j = \lambda_k \neq 0 \\ 0 & \text{otherwise} \end{cases}$$